Technical Notes

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Stagnation Streamline Turbulence Revisited

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Introduction

TAULBEE and Tran[1] find that the predictions of the k-ϵ theory of turbulence applied to the dividing streamline in a flow approaching a bluff body are in significant error and propose a Reynolds stress closure as a remedy. Our purpose is to show that if the empirical constants in the k-ϵ theory are adjusted to account for the two relevant regimes in this flow, namely far from the body where standard values prevail and close to the body where the two constants must be equal as pointed out by Strahle[2] in 1985, then the predictions in question are in good agreement with experimental data at least insofar as the turbulent kinetic energy is concerned. As a consequence calculations of the shear layer on a bluff body based on the k-ϵ theory according to Strahle et al.[3] are made consistent with the same theory applied to the approaching turbulent stream.

The flow under consideration is shown schematically in Fig. 1. Far from a two-dimensional stagnation point—the origin of an x-y coordinate system—there exists decaying turbulence with a uniform velocity $-V_0$. As the stagnation point is approached, the mean rate of strain field, compression in the y direction and extension in the x direction leads to an increase in the turbulent kinetic energy. Contiguous with the body is a shear layer, the second regime in the present discussion.

As noted by Strahle,[2] the existence of dissimilar length scales in the two regimes suggests application of the ideas of matched asymptotic expansions. To this end we introduce two parameters, the mean rate of strain $a \equiv (\partial U/\partial x_{y=0}) = -(\partial V/\partial y_{y=0})$, the turbulent kinetic energy just external to the shear layer k_∞, and two independent variables $\eta \equiv ay/k_\infty^{1/2}$ and $\bar\eta \equiv \frac{1}{2}ay/\bar V_0$ appropriate for the two regimes of the flow. If the bluff body is a circular cylinder, $a = 2\bar V_0/R$ so that $\bar V_0/R$ can be used instead of $\bar a$, but we retain the latter as a prime parameter since the analysis of the shear layer is carried out more naturally in terms of k_∞ and a. The length scale of the two regimes cited earlier are $k_\infty^{1/2}/a$, characterizing the shear layer and V_0/a, characterizing the flow approaching the bluff body. We deal with the practically interesting case of weak turbulence such that $k_\infty^{1/2}/V_0 \ll 1$, and for reasons discussed by Taulbee and Tran[1] with flows whose turbulence length scale is small compared with R.

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For present purposes, we are concerned only with the outer solution, that considered by Taulbee and Tran,[1] but we must recognize the need to match such a solution to that for the shear layer. Matching requires that $\bar\eta \to 0$ and $\eta \to \infty$. Strahle[2] argues that if $\eta \to \infty$, the two empirical constants in the k-ϵ theory, $c_{\epsilon 1}$ and $c_{\epsilon 2}$ following usual notation, must be equal within the shear layer. However, far from the stagnation point, the two constants must retain their standard values in order to describe decaying turbulance. Accordingly, to accommodate these requirements we take

$$c_{\epsilon 1} = c_{\epsilon 1\infty} + (c_\epsilon - c_{\epsilon 1\infty})e^{-\alpha\bar\eta} \tag{1a}$$

$$c_{\epsilon 2} = c_{\epsilon 2\infty} + (c_\epsilon - c_{\epsilon 2\infty})e^{-\alpha\bar\eta} \tag{1b}$$

where $c_{\epsilon 1\infty} = 1.44$ and $c_{\epsilon 2\infty} = 1.92$, i.e., standard values, and c_ϵ is the common value within and just external to the shear layer. Both c_ϵ and α are at our disposal to achieve agreement between prediction and experiment in the flow approaching the body.

With the exception of Eqs. (1), our analysis follows closely that of Taulbee and Tran.[1] Thus we describe the mean velocity components in terms of the potential flow approaching a circular cylinder so that

$$U = \frac{ax}{(1+\bar\eta)^3}$$

$$V = -V_0\left[1 - \frac{1}{(1+\bar\eta)^2}\right] \tag{2}$$

and the k-ϵ equations become:

$$-\left[1 - \frac{1}{(1+\bar\eta)^2}\right]\frac{\bar K'}{2} = 4c_\mu \frac{\bar K^2}{\bar E}\frac{1}{(1+\bar\eta)^6} - \bar E$$

$$-\left[1 - \frac{1}{(1+\bar\eta)^2}\right]\frac{\bar E'}{\bar K} = \frac{\bar E}{\bar K}\left[4c_{\epsilon 1}c_\mu \frac{\bar K^2}{\bar E}\frac{1}{(1+\bar\eta)^6} - c_{\epsilon 2}\bar E\right] \tag{3}$$

where $\bar K \equiv k/k_\infty$, $\bar E \equiv \epsilon/(ak_\infty)$ are dimensionless dependent variables and where the prime denotes differentiation with respect to $\bar\eta$. Equation (3) is to be solved subject to the initial conditions $\bar K(0) = 1$ and $\bar E(0) = 2(c_\mu)^{1/2}$, conditions consistent with matching such solutions with those for the the shear layer as $\eta \to \infty$ and with a balance of turbulence production and dissipation at the matching layer. It is worth noting that as $\bar\eta \to \infty$ these equations become

$$\bar K' \approx 2\bar E \qquad \bar E' \approx 2c_{\epsilon 2\infty}\frac{\bar E^2}{\bar K} \tag{4}$$

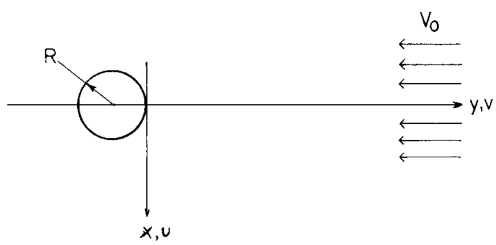

Fig. 1 Schematic representation of the flow.

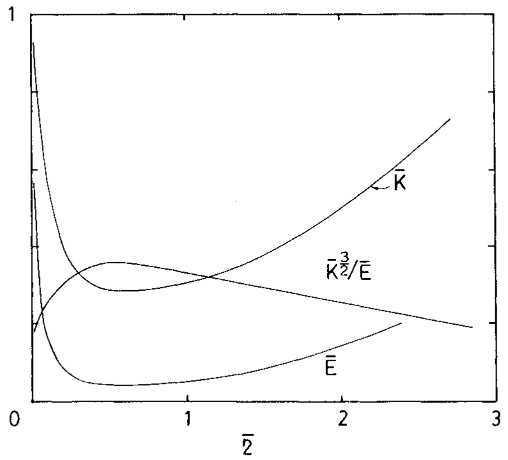

Fig. 2 Distributions of the dimensionless turbulent kinetic energy, viscous dissipation, and turbulent length scale.

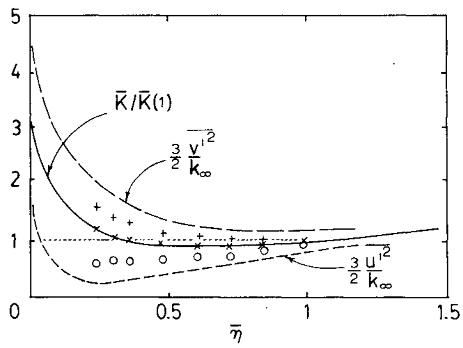

Fig. 3 Comparison with the experimental data of Ref. 4.

i.e., the equations for grid turbulence in the present coordinate system. If the grid is located at $\bar{\eta} = \bar{\eta}_g$, then Eqs. (4) yield solutions

$$\frac{1}{\bar{K}} = A_1(\bar{\eta}_g - \bar{\eta}) \qquad \frac{1}{\bar{E}} = A_2(\bar{\eta}_g - \bar{\eta})^2 \tag{5}$$

provided $c_{\epsilon 2\infty} = 2$. Here A_1 and A_2 are arbitrary constants. Equation (5) describe the idealized decay of turbulence downstream of a grid and are only slighly modified if $c_{\epsilon 2\infty}$ is assigned its standard value.

If the turbulent kinetic energy is known, the individual contributions thereto can be determined from the general turbulent stress-mean rate of strain relation for gradient transport. Thus we have

$$\overline{u'^2} = \frac{2}{3}\left(k - 3c_\mu \frac{k^2}{\epsilon}\frac{\partial U}{\partial x}\right) = \frac{2}{3}k_\infty \bar{K}\left[1 - 3c_\mu \frac{\bar{K}}{\bar{E}}\frac{1}{(1+\bar{\eta})^3}\right]$$

$$\overline{v'^2} = \frac{2}{3}\left(k - 3c_\mu \frac{k^2}{\epsilon}\frac{\partial V}{\partial y}\right) = \frac{2}{3}k_\infty \bar{K}\left[1 + 3c_\mu \frac{\bar{K}}{\bar{E}}\frac{1}{(1+\bar{\eta})^3}\right] \tag{6}$$

Our results are relatively sensitive to c_ϵ but insensitive to α. Thus, we take $\alpha = 1$ and find from numerical experimentation that an appropriate value for the former is $c_\epsilon = 2.27$. In Fig. 2 we show the distributions of $\bar{K}$, $\bar{E}$, and the ratio $\bar{K}^{3/2}/\bar{E}$, a dimensionless turbulence length scale, obtained with these values. With decreasing values of $\bar{\eta}$, we see, as expected, an initial decay of the turbulence and a corresponding initial increase in the turbulence length scale and a subsequent increase and decrease, respectively, under the influence of the rate of strain associated with the bluff body.

To compare these results with the data for the flow upstream of a circular cylinder given by Taulbee and Tran[1] as taken from Hijikata et al.[4], we recast the results in Fig. 2 in terms of $\bar{K}/\bar{K}(1)$ as shown in Fig. 3. The satisfactory agreement with respect to the turbulent kinetic energy is noted but, as suggested by Taulbee and Tran,[1] the distributions of the individual intensities are only qualitatively correct.

Several comments stemming from private communications and regarding this finding are indicated. Pope has pointed out to us that the failure of the standard k-ϵ theory for stagnating turbulence reflects an inaccurate description of the production process whereby isotropic turbulence under the influence of mean rates of compression and extension becomes anisotropic, an inaccuracy leading to the rapid increase in the turbulent kinetic energy found by Taulbee and Tran.[1] As noted by these authors, the Reynolds stress theory overcomes this failure and provides a more accurate description of this process. An alternative remedy is provided by Hijikita et al,[4] who modify the k-ϵ theory by introducing an equation accounting for the anisotropy of the $\overline{u'^2}$ and $\overline{v'^2}$ fluctuations. Our rehabilitation of the standard k-ϵ theory by the adjustment of the empirical constants via Eqs. (1) is recognized as an ad hoc remedy for this shortcoming. Although Taulbee has questioned the legitimacy of our remedy, it does provide a rational basis for application of the k-ϵ theory to the description of the shear layer as reported by Strahle et al.[3] Since we have selected c_ϵ on the basis of basis of data sufficiently close to the cylinder so that $v \approx -ay$, it should apply with an accuracy adequate for most purposes to other flows involving stagnating turbulence. Strahle and co-workers in a forthcoming publication have adapted a suggestion of Leschziner and Rodi,[5] which involves a modification of the dissipation equation providing an alternate means of matching shear layer and outer flow solutions. Finally, Peters has pointed out to us that since the production term in Eqs. (3) vanishes as $\bar{\eta} \to \infty$, it may be more appropriate to set the two constants in the dissipation equation equal everywhere in the approaching stream and to eliminate thereby the bridging function of Eqs. (1). We have not examined the implications of these two modifications, relative to the analysis of the approaching turbulent stream.

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